Learning Module Number 6

Beam Design by Elastic and Inelastic Analyses

Overview

The design of a laterally braced beam of compact section is investigated using elastic and inelastic analyses. In addition to observing differences in load carrying capacity and efficiency, force and moment redistribution as a result of member yielding is explored.

Learning Objectives

- Compare designs of a laterally braced beam of compact section using elastic and inelastic analyses.
- Observe the limitations of using elastic analysis and the benefits of employing inelastic analysis in the design of continuous beams.
- Employ the 9/10^{ths} moment redistribution allowance permitted in Section B3.3 the AISC *Specification for Structural Steel Buildings* (2016).
- Investigate force and moment redistribution that occurs as a result of member yielding.

Method

Prepare a computational model of the continuous three-member system shown in Fig. 1. Assume that all three members are compact wide-flange sections of A992 steel, oriented for major-axis bending, and fully braced out-of-plane (L_b = 0). Neglect the self-weight of the members.

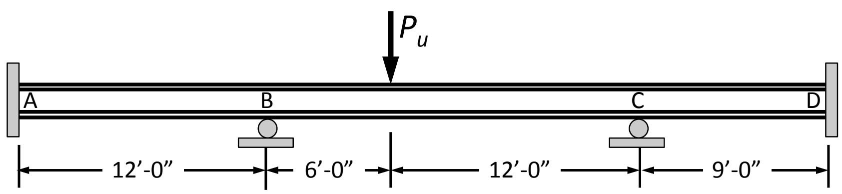

Figure 1.

According to Chapter F and Appendix 1 of the AISC *Specification for Structural Steel Buildings* (2016), the maximum moment in any of the beams comprising this system may not exceed the design plastic moment capacity, $M_u \leq \phi_b M_p$, in which ϕ_b = 0.9 and $M_p = ZF_y$, with Z = plastic section modulus and F_y = yield strength. Perform the case studies defined below for the following design methods:

a. First-order elastic analysis and <u>not</u> taking into account the 9/10^{ths} moment redistribution permitted in Section B3.3 the AISC *Specification for Structural Steel Buildings* (2016).
b. First-order elastic analysis and taking into account the 9/10^{ths} moment redistribution permitted in Section B3.3 the AISC *Specification for Structural Steel Buildings* (2016).
c. First-order inelastic analysis that according to Appendix 1 of AISC *Specification for Structural Steel Buildings* (2016) permits "the redistribution of member and connection forces and moments as a result of localized yielding."

Case studies:

1) Given that all members are W16x31, determine the largest value for P_u that is permitted by each of the above design methods. For simplicity, start with P = 100 kips in the computational model.
2) Given that member AB is a W27x84, member BC is a W16x31, and member CD is a W18x35, determine the largest value for P_u that is permitted by each of the above design methods. Again, start with P = 100 kips
3) Assuming that all three members must be the same compact wide-flange section, determine the least-weight section for each design method given that P_u=200 kips.
4) Assuming member sizes may vary (but must be compact wide-flange sections), determine the least weight system for each design method given that P_u=200 kips.

For each of the above studies, also compute the strength-to-weight ratio (P_u/Wt) of the systems designed. Record results in Table 1. For all inelastic analyses, record additional data related to the plastic hinge sequence in Table 2.

Table 1.

	AISC Design Method	Member Lengths and Sizes AB 12'-0"	BC 18'-0"	CD 9'-0"	P_u kips	Ratio $P_u/P_{u,inelastic}$	Weight kips	Ratio P_u / Wt
Case 1	a. Elastic	W16x31	W16x31	W16x31			1.209	
	b. Elastic 9/10th	W16x31	W16x31	W16x31			1.209	
	c. Inelastic	W16x31	W16x31	W16x31		1.0	1.209	
Case 2	a. Elastic	W27x84	W16x31	W18x35			1.881	
	b. Elastic 9/10th	W27x84	W16x31	W18x35			1.881	
	c. Inelastic	W27x84	W16x31	W18x35		1.0	1.881	
Case 3	a. Elastic				200	-		
	b. Elastic 9/10th				200	-		
	c. Inelastic				200	-		
Case 4	a. Elastic				200	-		
	b. Elastic 9/10th				200	-		
	c. Inelastic				200	-		

Table 2.

Case	P kips	Plastic Hinge Sequence 1st Hinge APL	1st Hinge Location	2nd Hinge APL	2nd Hinge Location	3rd Hinge APL	3rd Hinge Location	Ratio APL_{max} / APL_{min}
1-c								
2-c								
3-c	200							
4-c	200							

APL = Applied Load Ratio (P x APL = Load at which plastic hinge formed)

Hints:

1) Suggested units are kips, inches, and ksi.
2) Given that the members are fully laterally braced, 2-dimensional (planar frame) analyses are recommended.
3) Results of first-order elastic analyses are directly proportional to the applied load; for example, doubling the applied load will result in doubling the internal forces and moments.
4) Per Section 1.3.3a of Appendix 1 of the AISC *Specification for Structural Steel Buildings* (2016), the material yield strength employed in the inelastic analyses should be defined as $0.9F_y$ (thereby assuring $M_u \leq \phi_b M_p$) and the material stiffness should be reduced to E = 0.9 x 29000 ksi.
5) Assume second-order effects are negligible.
6) Given that the strength limit state in this study will always be controlled by the formation of a plastic mechanism, the computational models do not need to account for the effects of initial imperfections or partial yielding accentuated by the presence of residual stresses.
7) For all analyses, be sure to confirm that none of the member internal moments exceed $\phi_b M_p$

MASTAN2 Details

Per Fig. 2, the following suggestions are for those employing MASTAN2 to calculate the above computational strengths:

- ✓ Plastic hinges can only form at element ends. Realizing that peak moments will occur at the load P and at the supports, only 4 elements are needed.

- ✓ The failure load P_u will be the product of the applied force *P* and the resulting Applied Load Ratio.
- ✓ For the first-order inelastic analyses, use the following options:
    - Planar frame analysis type
    - Simple-step solution scheme
    - Load increment size of 0.1
    - Maximum number of increments set to 100
    - Maximum applied load ratio set to 10
- ✓ If the analysis pauses and indicates that a significant change in deformations is detected, this means that a plastic mechanism has formed. There is no need to continue the analyses.
- ✓ The deformed shape will include plastic hinge locations and sequence.
- ✓ Response curves may be prepared using MASTAN2's MSAPlot feature.

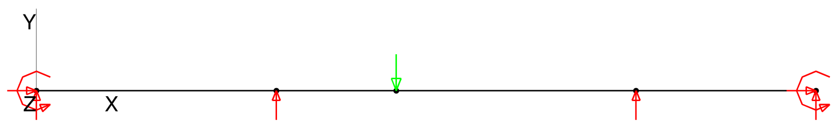

Figure 2. MASTAN2 model.

Questions

1) What level of additional load capacity was realized by employing inelastic analysis in Cases 1 and 2? In your opinion, is this significant and worth the additional effort a completing an inelastic analysis?
2) What level of additional efficiency (P_u/Wt) was realized by employing inelastic analysis in Cases 3 and 4? In your opinion, is this significant and worth the additional effort a completing an inelastic analysis?
3) What serviceability limit state may control the design of this system, and thereby potentially eliminate the benefits of employing an inelastic analysis?
4) Engineers have often stated that the design of compact fully laterally braced beams by elastic analysis (without the 9/10th's clause) can be defined as a process of designing for the first plastic hinge. Based on the results of this study, do you agree or disagree? Justify your response.
5) For the four cases explored in this study, does the elastic design method with the 9/10th's clause provide conservative or unconservative results when compared to design by inelastic analysis? Please describe any moment distributions (diagrams) for which this clause should be used cautiously.
6) For this system and given applied loading, is it possible for a plastic hinge to form at any point along the span that does not include either the location of the concentrated load P or support points A, B, C, and D. Justify you answer.
7) For this system and given applied loading, is it possible for a plastic hinge to ever form at support points A and D? Justify your response.
8) Of what interest to the designer is the plastic hinge sequence (APL and location) and ratio APL_{max}/APL_{min} given in Table 2?
9) If the beam is supported in the same manner but is not continuous at points B and C, what would the maximum concentrated load P_u be for cases 1 and 2? What can you conclude about the benefits of inelastic analysis when used to design a statically determinate system?
10) Why is it essential to the inelastic analysis design method that the cross-section elements be compact? (Hint: See Fig. C-A-1.2 and the corresponding text in the AISC commentary to Appendix 1.3.2b)

More Fun with Computational Analysis!

1) For the inelastic analysis completed in Case 2c, prepare the following plots:
    a. A curve with the magnitude of the vertical displacement (in.) at the concentrated load *P* on the abscissa and the magnitude of the applied concentrated load *P* (kips) on the ordinate. For clarity, you may want to continue the inelastic analysis for one additional load step after the plastic mechanism has formed. Indicate on the curve the sequence of the plastic hinges. Use this force-displacement plot to approximate the percent reduction in stiffness, 100% x $(k_o\text{-}k_i)/k_o$, that

corresponds to the formation of each plastic hinge. Approximate the stiffness k_i as the slope of the curve and k_o as the initial slope of the curve.

b. Three curves that plot the magnitude of the bending moment (kip-in) at each plastic hinge as the abscissa and the magnitude of the applied concentrated load P (kips) on the ordinate. Do these curves clearly show the moment redistribution that has taken place? Why do all three curves terminate at the same point?

2) Repeat the above study for a uniform load w that is distributed over all three members. For cases 3 and 4, let w_u = 0.1 kip/in.
3) Repeat the above study for three equal concentrated loads P located at the mid-span of the members. For cases 3 and 4, let P_u = 100 kip.

Additional Resources

MS Excel spreadsheet: *6_BeamDesignElasticInelasticAnalysis.xlsx*

MASTAN2 – LM6 Tutorial Video [11 min]:

http://www.youtube.com/watch?v=qWyh_lcDTBs

MASTAN2 - How to plot response curves with MSAPlot [3 min]:

http://www.youtube.com/watch?v=vS67MT0M1PQ

AISC *Specification for Structural Steel Buildings and Commentary* (2016):

https://www.aisc.org/publications/steel-standards/ - 30666

MASTAN2 software:

http://www.mastan2.com/