Learning Module Number 8

Strength of Beam-Columns

Overview

The strength of beam-columns is investigated for various combinations of axial force and bending moment. Both major- and minor-axis flexure of steel wide-flange sections with compact elements are investigated. Nominal strength curves per requirements of Chapter H of the AISC *Specification for Structural Steel Buildings* (2010) are compared with results obtained using second-order inelastic analyses. Strength limit states defined by elastic/inelastic flexural and lateral torsional buckling as well as full yielding of the cross-section (plastic hinge) are studied.

Learning Objectives

- Observe the strength limit state behavior of beam-columns, which includes the range of full yielding of the cross-section to elastic/inelastic flexural and lateral torsional buckling.
- Prepare interaction curves that plot member axial strength versus flexural strength.
- Compare results of the AISC interaction equations with results from computational analyses that account for partial yielding (accentuated by the presence of residual stresses) and initial imperfections in geometry.

Method

A W14x53 (A992 steel) member with an unbraced length of 15’-0” will be subjected to various combinations of axial compressive force and major- or minor-axis bending (Fig. 1). The following two studies are to completed:

1) AISC Strength
   Prepare a three-dimensional computational model of the beam-column shown in Fig. 1. Given that the AISC equations for computing nominal strengths P_n and M_n account for initial imperfections (out-of-straightness) and partial yielding accentuated by the presence of thermal residual stresses, the computation model and analyses should not include these factors. In this regard, second-order elastic analyses will be employed. The following steps are suggested for completing Tables 1a and 1b:
   a) Reference Chs. E and F of the AISC *Specification for Structural Steel Buildings* (2010) to confirm the numerical values given for nominal minor-axis compressive strength P_n and major- and minor-axis flexural strengths M_n’s.
   b) Confirm the given axial yield (squash) load P_y and plastic moment capacities M_p’s.
   c) Noting that nearly half of the results in the tables have been provided, please be sure to confirm the values given in several of these completed rows. M_u is the maximum moment occurring within the member’s span. Because this study is intended to compare nominal strengths, the resistance ϕ factors given in the AISC interaction Eq. H1-1a/b should <u>not</u> be included.
   d) Complete the remaining rows of Tables 1a and 1b. The given combinations of axial force and bending moments at the member ends have been purposely chosen to result in unity values for the AISC interaction Eq. H1-1a/b, thereby representing the maximum nominal strengths for this member as permitted by the AISC Specification.
2) Computational Strength
   Modify the above three-dimensional computational model to include an initial out-of-straightness of $L/1000$ in a direction normal to the plane of the web (i.e. that will result in minor-axis bending when the member is subject to axial force). Nominal strengths will be determined employing second-order inelastic analyses that also account for partial yielding accentuated by the presence of thermal residual stresses. The following steps are suggested for completing Tables 2a and 2b:
   a) Confirm the given axial yield (squash) load P_y and plastic moment capacities M_p’s.
   b) Noting that nearly half of the results for this study are provided, please be sure to confirm the numbers given in several of these completed rows. ALR_{ult} is the maximum applied load ratio at which the analysis indicates the member has reached a limit state of strength.

c) Complete the remaining rows of Tables 2a and 2b. The given combinations of axial force and bending moments at the member ends are consistent with those assessed in Tables 1a and 1b. In this regard, comparing the ALR_{ult} to unity will indicate whether or not the AISC interaction equation is conservative. In completing Table 2b, it is important that the direction of minor-axis bending moments should compliment (not offset) the impact of the given initial out-of-straightness combined with the axial compressive force. Be sure to view the deformed shape at the computed ultimate strength and record the failure mode as either full yielding (plastic hinge) of the cross-section, or elastic/inelastic flexural and/or lateral torsional buckling.

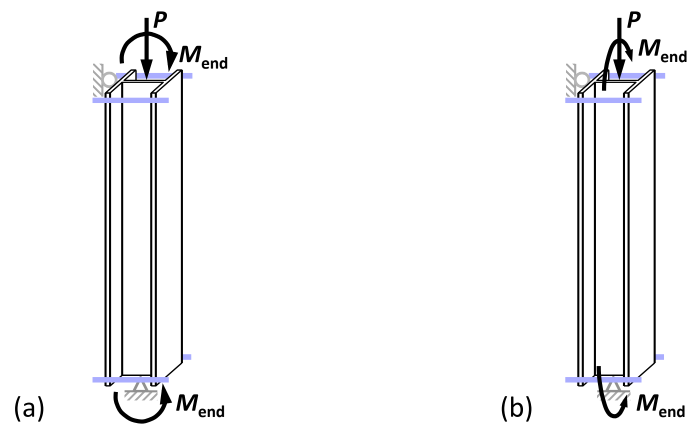

Figure 1. Beam-column subject to major- and minor-axis flexure.

Hints:
1) Suggested units are kips, inches, and ksi.
2) Maintain two computational models, one without imperfections and one with imperfections. The initial imperfection of L/1000 is such that the member is initially bent about its minor-axis.
3) 3-Dimensional (space frame) analyses are required. Support conditions at the member ends should include all translation degrees of freedom restrained with the exception of longitudinal translation at one end of the member. Torsional degrees of freedom (rotation about the longitudinal axis) at both member ends should also be restrained. Warping should be modeled as continuous along the span length and free at the member ends.
4) Member ends should be loaded with equal and opposite major- or minor-axis bending moments.
5) Do not include the self-weight of the member.

MASTAN2 Details

Per Fig. 2, the following suggestions are for those employing MASTAN2 to calculate the above computational strengths:

- ✓ Subdivide the member into 8 elements.
- ✓ Initial imperfections (as needed) can be included by either extensive use of the *Move Node* option, or much more easily by "permanently bending" the member through the combined use of either a buckling analysis or lateral load analysis, and MASTAN2's post-processing option *Results-Update Geometry*.
- ✓ Because it may be difficult to observe twist when working with one-dimensional line elements, it is suggested that a few additional elements be added at the mid-span of the member that are perpendicular to its longitudinal axis. Given that these elements should not resist any of the applied moments, their section properties only need to be non-zero.
- ✓ For the AISC Strength study, employ second-order elastic analyses with:
  - Space frame analysis type
  - Predictor-corrector solution scheme
  - Load increment size of 0.1

  - Maximum number of increments set to 10
  - Maximum applied load ratio set to 1.0
- ✓ For the Computational Strength study, employ second-order inelastic analyses with:
  - Space frame analysis type
  - Predictor-corrector solution scheme
  - Load increment size of 0.01
  - Maximum number of increments set to 10000
  - Maximum applied load ratio set to 10
  - Modulus set to E_{tm} (account for partial yielding/residual stresses)
- ✓ If the analysis pauses and indicates that a significant change in deformations is detected, this means that a plastic mechanism has formed. There is no need to continue the analyses.
- ✓ Warping resistance to torsion can be modeled along the member span by using MASTAN2's option under *Geometry* > *Define Connections* > *Torsion* and setting the warping restraint at both ends of all elements to "Continuous."

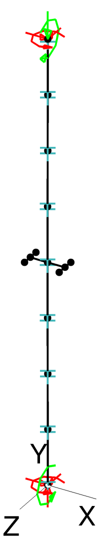

Figure 2. MASTAN2 model of beam-column.

Table 1a. AISC Strength study for axial compression plus major-axis bending (Fig. 1a).

P_n =	410	kips	$P_y = A_g \times F_y$ =	780	Kips
M_n =	3486	kip-in	$M_p = Z \times F_y$ =	4355	in-kips
P (kips)	M_{end} (kip-in)	M_u (kip-in)	Eq. H1-1a/b[1]	M_{end} / M_p	P/ P_y
0	3486	3486	1.00	0.80	0.00
41	3277				
82	3070	3136	1.00	0.70	0.11
123	2659				
164	2254	2353	1.00	0.52	0.21
205	1857				
246	1470	1568	1.00	0.34	0.32
287	1090				
328	718	783.3	1.00	0.16	0.42
369	355				
410	0	0	1.00	0.00	0.53

[1] Based on nominal strength (ϕ's=1.0)

Table 1b. AISC Strength study for axial compression plus minor-axis bending (Fig. 1b).

P_n =	410	kips	$P_y = A_g$ x F_y =	780	kips
M_n =	1100	kip-in	$M_p = Z$ x F_y =	1100	in-kips
P_u (kips)	M_{end} (kip-in)	M_u (kip-in)	Eq. H1-1a/b[2]	M_{end} / M_p	P_u / P_y
0	1100	1100	1.00	1.00	0.00
41	943				
82	800	989.6	1.00	0.73	0.11
123	622				
164	468	742.7	1.00	0.43	0.21
205	338				
246	230	495.1	1.00	0.21	0.32
287	143				
328	77	247.4	1.00	0.07	0.42
369	29				
410	0	0	1.00	0.00	0.53

Table 2a. Computational Strength study for axial compression plus major-axis bending (Fig. 1a).

Second-Order Inelastic Analysis (E = 29,000 ksi, F_y = 50 ksi)				$P_y = A_g$ x F_y =	780	kips
Partial yielding/σ_{res} included and $\delta_o = L/1000$ = ___________in.				$M_p = Z$ x F_y =	4355	in-kips
P (kips)	M_{end} (kip-in)	ALR_{ult}	ALR_{ult} x P	ALR_{ult} x M_{end}	ALR_{ult} x M_{end}/M_p	ALR_{ult} x P/P_y
0	3486	0.89	0	3103	0.71	0.00
41	3277					
82	3070	0.88	72.2	2702	0.62	0.09
123	2659					
164	2254	0.98	161	2209	0.51	0.21
205	1857					
246	1470	1.05	258	1544	0.35	0.33
287	1090					
328	718	1.04	341	747	0.17	0.44
369	355					
410	0	0.89	365	0	0.00	0.47

Table 2b. Computational Strength study for axial compression plus minor-axis bending (Fig. 1b).

Second-Order Inelastic Analysis (E = 29,000 ksi, F_y = 50 ksi)				$P_y = A_g$ x F_y =	780	kips
Partial yielding/σ_{res} included and $\delta_o = L/1000$ = ___________in.				$M_p = Z$ x F_y =	1100	in-kips
P (kips)	M_{end} (kip-in)	ALR_{ult}	ALR_{ult} x P	ALR_{ult} x M_{end}	ALR_{ult} x M_{end}/M_p	ALR_{ult} x P/P_y
0	1100	1.00	0.0	1100	1.00	0.00
41	943					
82	800	0.97	79.5	776	0.71	0.10
123	622					
164	468	0.98	161	459	0.42	0.21
205	338					
246	230	0.96	236	221	0.20	0.30
287	143					
328	77	0.95	312	73.2	0.07	0.40
369	29					
410	0	0.89	365	0	0.00	0.47

[2] Based on nominal strength (ϕ's=1.0)

Questions

1) In Tables 1a and 1b, compute the ratios of maximum moments M_u to the end moments M_{end}. In general, what can be concluded regarding second-order effects and major- versus minor-axis bending?
2) Prepare a single plot that includes two curves defined by the highlighted two-rightmost columns of Tables 1a (AISC Strength) and 2a (Computational Strength), with the limiting normalized end moments M/M_p as the abscissa and the limiting normalized axial force P/P_y as the ordinate. In general, how does the rather simple form of the AISC interaction perform in adequately defining the strength of a beam-column subject to the combination of axial force and major-axis flexure? In responding, feel free to reference the ALR_{ult} values given in Table 2a.
3) Using the highlighted two-rightmost columns of Tables 1b (AISC Strength) and 2b (Computational Strength), repeat the previous question for the case of a beam-column subject to the combination of an axial compressive force and minor-axis flexure.
4) Why is the AISC curve presented in Question 3 much more nonlinear in the higher axial force range (P/P_y > 0.2) than the AISC curve presented in Question 2?
5) Using terms such as full yielding (plastic hinge) of the cross-section, or elastic/inelastic flexural and/or lateral torsional buckling, please describe the failure modes for:
    a. Major-axis bending and no axial force
    b. Axial force and no bending
    c. Minor-axis bending and no axial force
    d. Transition from significant major-axis bending to negligible major-axis bending
    e. Transition from significant minor-axis bending to negligible minor-axis bending

More Fun with Computational Analysis!

1) Repeat the above Computational Strength study assuming that warping is still continuous along the span but fixed at the member ends. Given that most beam-columns are continuous with other members in the system (i.e. warping conditions at member ends are closer to fixed than free), comment on the conservatism of the AISC interaction equation (which does not account for warping continuity at member ends).
2) Repeat the above AISC and Computational Strength studies for the single case of compression plus bi-axial bending. Use an axial force of P = 110 kips and equal/opposite major- and minor-axis end moments of 1480 kip-in and 210 kip-in, respectively. What is the ALR_{ult} and what does this indicate with regard to the AISC interaction equation for cases of compression plus bi-axial bending?
3) In the above studies, the initial imperfection was only included in a direction normal to the plane of the web. Explore a few cases to determine the impact of including an additional initial imperfection in the plane of the web. Provide a plausible explanation for what was observed for these cases.
4) Repeat the above studies for a W14x176 (A992 steel) with an unbraced length of 32'-0". Warning – there is a significant amount of trial and error involved in producing force and end moment combinations that result in unity values for the AISC interaction equation.

Additional Resources

MS Excel spreadsheet: *8_StrengthOfBeamColumns.xlsx*
MASTAN2 – LM8 Tutorial Video [15 min]:
    http://www.youtube.com/watch?v=pa6nBKD32Lg
MASTAN2 - How to include warping resistance [1 min]:
    http://www.youtube.com/watch?v=ttoVaiEnn0M
MASTAN2 - How to include an initial imperfection (member out-of-straightness) [4 min]:
    http://www.youtube.com/watch?v=v3ON1faDSZo
MASTAN2 - How to account for partial yielding accentuated by residual stresses [1 min]:
    http://www.youtube.com/watch?v=m8ZXM02Cbu4
AISC *Specification for Structural Steel Buildings and Commentary* (2010):
    http://www.aisc.org/content.aspx?id=2884
MASTAN2 software:
    http://www.mastan2.com/